Tangential momentum accommodation coefficient in a microtube

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Abstract :

Experimental investigations of isothermal steady flows for various gases have been carried out in a silica micro tube. The study is focused on the mass flow rate measurements of these flows in slip regime, using a suitable powerful platform. First we analyse, for each gas, the pertinence of a first or second order continuum treatment; then we deduce from experiments, using the appropriate treatment, the tangential momentum accommodation coefficient (TMAC) of each gas. The TMAC obtained for the various pairs of gas (nitrogen, argon, helium)/surface (fused silica) exclude a full diffuse reflection.

Résumé :

Une campagne expérimentale d'écoulement gazeux (azote, argon et helium) a été conduite dans un microtube de silice. Les écoulements expérimentaux sont obtenus par un gradient de pression dans des conditions isothermes. Ces expériences ont été réalisées en régime de glissement, elles ont necessité le développement d'une plateforme expérimentale performante pour mesurer de très faibles débits massiques. Une approche analytique basée sur les équations de Navier-Stokes (NS) avec des conditions limites du premier ou du second ordre selon leur pertinence ont été utilisées pour déterminer les coefficients d'accommodation (TMAC). Les coefficients obtenus pour les différentes paires gaz(azote, argon, helium)/surface (silice) excluent une réflexion totalement diffuse à la surface.

Key-words :

microflow, tangential momentum accommodation coefficient, slip coefficient

1 Introduction

The Micro-Electro-Mechanical-Systems open a new area in the rarefied gas experiments. Indeed, since the early eighties and the beginning of the MEMS, a lot of micro devices were designed to study gas micro flows. But channel geometries involving a rectangular (or trapezoidal) cross section have been privileged until to now (Pong et al. (1994)) - (Maurer et al. (2003)). In this work we present a gas micro flow study based on micro tube experiments. In this geometry the experiments are rare, only four different experiments involving rarefied gas flows in tubes or micro tubes have been undertaken in the last fifty years: using a tube with $3.64\,cm$ of diameter in Dong W. (1956); with a package of 10 tubes of a mean radius equal to $199.7\,\mu m$ and also in a package of 100 tubes of a mean radius equal to $50\,\mu m$ in Porodnov B.T. et al (1974); and finally with a package of 50 tubes with a radius of $25\,\mu m$ in Lalonde P. (2001). Thus, nobody performed experiments in a single micro tube characterized by a diameter precisely known. One of the reasons of this lack is due to the difficulty of measuring mass flow rates so weak as those flowing in a single micro tube (smaller than $10^{-10}kg/s$): in point of fact, in a tube the mass flow rate can be from 3 to 100 times lower than that found in a rectangular channel for the same inlet/outlet pressure ratio, with the same streamwise length, and with the same small critic geometric dimension, *i.e.* for the same values of the Knudsen number. Indeed,

in micro tube the small dimension is necessary the diameter, involved in the cross sections by its square power, while in micro channel only one dimension of the rectangular section is necessary small: thus using a large width, *i.e.* a small height-to-with ratio it is possible to increase largely the flow rate without changing the Knudsen number. Let us add that, for same basic geometric reasons, the dynamics of the flow in the tubes is in any case a two-dimensional problem, contrarily to that occurs in the rectangular channels where the problem becomes three-dimensional when the height-to-width ratio is not small enough. Thus some experiments exist concerning the TMAC in MEMS but, according to our previous remarks, they occurred in rectangular channel geometries (Arkilic et al. (2001)), (Colin et al. (2004)) and (Maurer et al. (2003)) or using several tubes in a package (Porodnov B.T. et al (1974)). In anyway these experiments remain very few numerous compared to those carried out in the molecular beam domain (Saxena S.C. et al. (1981)) which in many cases did not concern really the same accommodation coefficient. Therefore the present determination appears of some scientific interest.

2 Experiments

The experimental method used in the present work in order to measure the mass flow rate through a micro tube involves the use of two constant volume tanks and so is denoted "constant-volume technique". This method requires very large tank volumes, much larger than the volume of the micro tube. Large tank sizes guarantee micro flow parameters independent of the time: although detectible (through their effects), the mass variations occurring in the tanks during the experiments should not call into question the stationary assumption. Thus, we have to fix a range for the maximal suitable pressure variations in the second tank, according to the inlet and outlet conditions. The experimental set up takes into account these constraints. The gas flows through a micro tube fixed between two tanks in which the pressures remain very close to constant values P_{in} and P_{out}, respectively. The maximum pressure variation in the second tank due to the gas flow through the microtube is fixed at $\pm 1\%$ of the tank pressure averaged over the duration of the experiment. This variation range means that the required experiment duration τ will vary from 5 minutes for the highest mass flow rate measured (10^{-9}kg/s) to about 90 minutes for the lowest ($10^{-13} kg/s$).

Here, we omit the detailed description of methodology and experimental set-up. The validity of the measurements and modus operandi were approved in Ewart et al. (2006). To determine the mass flow rate we will use the registered data for the pressure at the different time instants. The flow stationary conditions physically justify the pressure rise interpolation by means of a linear fitting function of time.

The usual evaluation of the measurement errors is applied and gives the following results: a full uncertainty on $\Delta Q_m/Q_m$ smaller than $\pm 4.5\%$, where the non-isothermal effects are previously evaluated as $\pm 2\%$; the uncertainty on the volume measure is $\pm 2\%$ and the error on coefficient of the linear fitting of the pressure measurements is $\pm 0.5\%$. Moreover, the leaks were estimated with two different tests as totally negligible Ewart et al. (2006).

3 Background theory

For many years, pressure-driven slip flows within ducts or channels have received considerable attention. Many formulations of analytical and semi-analytical solutions have been presented (Karniadakis and Beskok (2002)). The analytical models derived from the Navier-Stokes equations or from other continuum equation systems require the use of the velocity slip boundary conditions. Several authors (Colin et al. (2004)), (Maurer et al. (2003)) have recently proposed

to use in this framework the velocity slip conditions of second-order according to the Knudsen number to take better into account the rarefied effects for the moderately rarefied gas flows.

In the hydrodynamic and slip regimes the flow through the micro tube have been intensively studied theoretically. Nevertheless the questions of the choice of appropriate boundary conditions (first or second order following the Knudsen number) and the question of the limit of validity of the continuum approach (in terms of the Knudsen number range) remains open question which are discussed below.

The flow analysis may be carried out with the Navier-Stokes equations, using slip boundary conditions. Assuming a second order boundary condition at the wall the slip velocity reads:

$$u_s = \pm A_1 \lambda \left(\frac{\partial u}{\partial r}\right)_w - A_2 \lambda^2 \left(\frac{1}{r}\left(\frac{\partial}{\partial r} r \frac{\partial u}{\partial r}\right)\right)_w, \tag{1}$$

where $\lambda = k_\lambda \frac{\mu}{P}\sqrt{2\mathcal{R}T}$ is the mean free path of the molecules which could be calculated using the hard sphere (HS) model (Chapman (1970)) where $k_\lambda = \sqrt{\pi}/2$, nevertheless, in this paper we used the variable hard sphere model (VHS) (Bird (1994)) more general than HS model. According to this model, coefficient k_λ is equal to $\frac{(7-2\omega)(5-2\omega)}{15\sqrt{\pi}}$, where ω, the viscosity index, depends on the type of gas: The coefficients A_1 and A_2 in (1) may be presented in the form:

$$A_1 = \sigma_p / k_\lambda, \qquad A_2 = \sigma_{2p} / k_\lambda^2, \tag{2}$$

where σ_p and σ_{2p} are the first and second velocity slip coefficients that are assumed independent of the interaction model used in the gas.

The non-dimensional mass flow rate S through the tube of diameter D, obtained from Navier-Stokes equations with the second order velocity slip condition (Graur et al. (2006)), reads

$$S = \dot{M} / \frac{\pi D^4 \Delta P P_m}{128 \mu \mathcal{R} T L} = 1 + 8 A_1 K n_m + 16 A_2 \frac{\mathcal{P}+1}{\mathcal{P}-1} \ln \mathcal{P} K n_m^2. \tag{3}$$

where $\dot{M}$ is the mass flow rate, $\Delta P = P_{in} - P_{out}$, $\mathcal{P} = P_{in}/P_{out}$, Kn_m is the mean Knudsen number, based on the mean pressure $P_m = 0.5(P_{in} + P_{out})$. The analytical expressions of the mass flow rate (2), (3) will be used for the calculation and the comparison with the appropriate measured values. We will use the measured values of the mass flow rate to obtain the slip velocity coefficients and the "experimental" TMAC.

4 Results and discussion

We have studied the flows of Argon, Nitrogen and Helium in slip regime where the mean Knudsen number varies from 0.02 to 0.3. Each experiment was carried out with different pressure ratios $\mathcal{P}= [3, 4, 5]$ between the tanks, excepted the N_2 ones which are limited to $\mathcal{P}= [3, 5]$. Figure 1 shows the experimental dimensionless mass flow rates (calculated according to (3)) for all the gases as a function of the mean Knudsen number.

In order to estimate the velocity slip coefficients the measured dimensionless mass flow rate was fitted with the first and second order polynomial form of the mean Knudsen number

$$S_f^{exp} = 1 + A_i^{exp} K n_m + B_i^{exp} K n_m^2, \qquad i = 1, 2 \tag{4}$$

as it was detailed in Maurer et al. (2003) using a non-linear least square Marquard-Levenberg algorithm. Experimental fitting coefficients A_i^{exp} and B_i^{exp}, where $i = 1, 2$ and $B_1^{exp} = 0$,

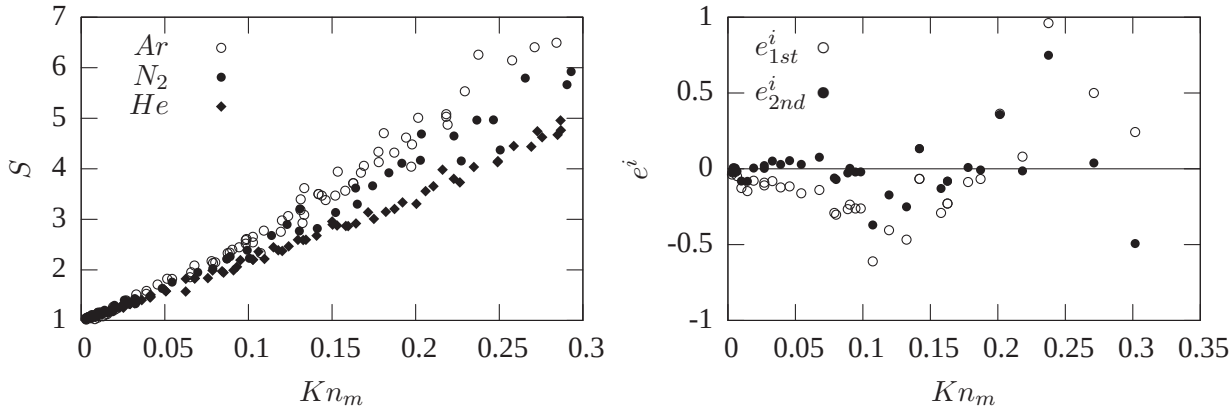

Figure 1: Dimensionless mass flow rate for N_2, Ar and He gases obtained according to (3) and Ar residuals for $\mathcal{P}_{Th}$= 5.

are calculated for all the gases and the uncertainty on these coefficients is calculated using the asymptotic standard error. These coefficients obtained for the pressure ratio $\mathcal{P} = 5$ are reported in Table 1. In order to analyze the respective pertinence of the first or the second order fitting for each gas, two additional parameters are calculated: the determination coefficient r^2 and the residual variance $s_r = \sqrt{\frac{1}{n-p}\sum e_i^2}$, where $e_i = S_i^{exp} - S_{f_i}^{exp}$ is the local difference between the measured and fitting values, and so represents the local fitting error; n is the number of points and p is the number of unknown coefficients of the fitting model. Analyzing the values of these two coefficients, given in Table 1 only for $\mathcal{P} = 5$, (the other $\mathcal{P}$ values give similar results for these coefficients) we find that the determination coefficients r^2 of Argon and Nitrogen are essentially more close to 1 for the second order fitting. For the Helium flow the second order coefficients r^2 is also more close to 1 than the first order one, even if the difference between the two orders is here less important. Moreover, the values of the squared residual sum are also smaller for the second order fitting in the case of all the gases. In order to supplement this analysis, the residuals e_i (fitting errors) are plotted as a function of the averaged Knudsen number for the three gases. As an example, the residuals of Argon are presented in Fig. 1. The analysis of the form of the residuals distribution shows that the residuals of second order fit are equi-distributed, whereas the residuals of the first order fit are largely negative from 0.003 to 0.2 on the Knudsen axis, which confirms the choice of the second order fitting as more pertinent for Argon flows. The same analysis of the the form of residuals is carried out for Nitrogen and Helium. From this analysis we may conclude that the second order fitting appears clearly as the most pertinent for Nitrogen and Argon flows and also for the Helium flow: even if as shown in Table 1 the relative weight of the second order coefficient is smaller for Helium as for the other gases. Thus, in the sequel of this paper, we will use the results of the second order fitting for all the gases.

From the comparison of the theoretical and experimental non-dimensional mass flow rate expressions (3), (4) the coefficients A_1 and A_2 from the velocity slip boundary condition (1) and respectively the slip coefficients σ_p and σ_{2p} may be found from the expressions:

$$A_1 = \frac{\sigma_p}{k_\lambda} = A^{exp}/8, \qquad B^{exp} = 16A_2\frac{\mathcal{P}+1}{\mathcal{P}-1}\ln\mathcal{P}, \qquad A_2 = \frac{\sigma_{2p}}{k_\lambda^2}. \tag{5}$$

The values of the these coefficients are given in Table 2. We also derived an experimental value of the accommodation coefficient using the Maxwell diffuse-specular scattering model.

$\mathcal{P}_{Th}$= 5	A_{1st}^{exp}	$s_{r_{1st}}$	$r_{r_{1st}}^2$	A_{2nd}^{exp}	B_{2nd}^{exp}	$s_{r_{2nd}}$	$r_{r_{2nd}}^2$
Nitrogen	15.494 ± 0.313	0.2115	0.9779	11.668 ± 0.967	16.626 ± 4.059	0.1931	0.9859
Argon	18.177 ± 0.391	0.2851	0.9691	13.218 ± 0.799	24.274 ± 3.698	0.1903	0.9863
Helium	12.991 ± 0.154	0.1315	0.9905	10.812 ± 0.371	9.156 ± 1.500	0.0876	0.9959

Table 1: Fitting parameters obtained from the experiments.

$\mathcal{P}_{Th}$	σ_p^{1st}	α_M^{1st}	σ_p^{2nd}	α_M^{2nd}	σ_{2p}^{2nd}	Kn_m
			Nitrogen			
5	1.415 ± 0.028	0.770 ± 0.010	1.066 ± 0.088	0.908 ± 0.041	0.231 ± 0.057	0.003-0.291
			Argon			
$5-4$	1.558 ± 0.021	0.725 ± 0.007	1.147 ± 0.042	0.871 ± 0.017	0.294 ± 0.029	0.003-0.302
			Helium			
$5-3$	1.252 ± 0.009	0.829 ± 0.004	1.052 ± 0.020	0.914 ± 0.009	0.148 ± 0.014	0.009-0.309

Table 2: σ_p^i and σ_{2p}^i experimental coefficients obtained from a polynomial fitting of first or second degree, α_M is obtained from the velocity slip coefficient using Maxwell relation.

The use of Maxwell's kernel for the gas-surface interaction gives the following value for the velocity slip coefficient, neglecting the Knudsen layer influence: $\sigma_p^M = \frac{\sqrt{\pi}}{2}\frac{2-\alpha}{\alpha}$. As well known, in the Maxwell kernel the same coefficient α may represent the energy accommodation as well as that of any momentum component. However, in isothermal slip regime it is usual and physically justified to identify α as the TMAC. In the case of a full accommodation ($\alpha = 1$) the theoretical coefficient σ_p^M is equal to 0.886. The different values of α are given in Table 2.

The previous data may be summarized as follows:

- in the investigated Knudsen range the relative weight of the second order effect (B_{2nd}/A_{2nd}) increases with the molecular mass and does not depend on the molecular internal structure (see Table 1).
- the TMAC deduced are strictly smaller than 1 excluding a complete diffuse reflection on the fused silica. The accommodation coefficient for Helium is significantly greater than the other gas coefficients.
- Table 3 shows a good agreement of the present values with other author experimental results if considering that the geometry of Arkilic et al. (2001), Colin et al. (2004), Maurer et al. (2003) was not circular, that the surface materials were generally not exactly the same (generally silica and silicon are both involved for a part in the channel shape), and that finally the pressure is generally not the same; moreover, certain authors derived the TMAC from a first order treatment.

	Nitrogen	Argon	Helium
α_{1st}^{exp}	0.77 ± 0.01	0.71 ± 0.01	0.82 ± 0.01
α_{2nd}^{exp}	0.91 ± 0.04	0.88 ± 0.03	0.91 ± 0.02
Porodnov B.T. et al (1974)	0.925 ± 0.014	0.927 ± 0.028	0.895 ± 0.004
Arkilic et al. (2001)	$0.81 - 0.96$	$0.7 - 1$	-
Maurer et al. (2003)	0.87 ± 0.03	-	0.91 ± 0.03
Colin et al. (2004)	0.93	-	0.93

Table 3: TMAC obtained from present experiments ($\mathcal{P}_{Th}$= 5) and by other authors.

- In order to study the detailed influence of geometry or pressure ratio on the TMAC more systematic experiments would be needed, even if in our experiments the value obtained for $\mathcal{P}$ with the Nitrogen and Argon seems to give results a little lower than other $\mathcal{P}$ values.

5 Conclusions

This work contributes to clarify the validity domains of slip regime modelling using first or second order boundary conditions. For the gases considered in the $0.003 - 0.3\ Kn$ range, in tube geometry, the second order fitting seems the most convenient. The TMAC determination leads to conclude that the He, Ar, and N_2 molecules are not reflected on silica surface following a full diffuse reflection. The Helium TMAC appears significantly greater than those of two other gases. To conclude on influence of inlet/outlet pressure ratio (or of geometry) on the accommodation process (for a same Knudsen number) would need more systematic experiments.

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